Water Resource Management Using Stochastic Multicriteria Population-Based Algorithms for Producing Alternatives

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Abstract

Water Resources Management (WRM) problems frequently possess inconsistent performance components and conflicting design requirements. Consequently, when solving complex WRM problems, it is often preferable to study a number of quantifiably good alternatives that encompass multiple, dissimilar perspectives. These alternatives need to satisfy the specified system performance criteria but be maximally different from each other in the decision space. The approach for creating maximally different sets of solutions is referred to as Modelling-to-Generate-Alternatives (MGA). Simulation-optimization approaches are frequently employed to solve computationally difficult problems containing significant stochastic uncertainties. This paper outlines a multicriteria MGA approach for WRM that can produce sets of maximally different alternatives for any simulation-optimization method that employs a population-based solution algorithm. This algorithmic approach is computationally efficient because it simultaneously produces the prescribed number of maximally different solution alternatives in a single computational run of the procedure. The efficacy of this stochastic multicriteria MGA approach for creating alternatives is demonstrated using a “real world” water resource management planning case.

Keywords: Metaheuristics; Modelling-to-generate-alternatives; Population-based algorithms; Simulation-Optimization; Water resources management

Introduction

Water resource managers have been challenged by difficult water allocation problems for many decades [1,2]. Implementing effective Water Resources Management (WRM) has proven to be notoriously antagonistic as inherent conflict between multiple industrial, agricultural, and municipal water-users has intensified. Declining water supplies together with increased population shifts have further diminished the inter-user relations. These aggravations exacerbate the hostilities when the natural conditions become more unpredictable as concern for water quantity and quality grows. Ill-conceived water allocation schemes can increase into more serious conflicts under detrimental river-flow and changing climatic conditions. In the past, increasing demand for water was satisfied by the development of new water sources. However, the significant economic and environmental costs associated with developing new water sources have rendered this tactic unsustainable. Unlimited expansion of water sources is no longer the primary focus of WRM. Instead, for optimum WRM, the objective has become the improvement of existing water management by shaping water allocation techniques in a more equitable, environmentally-benign, and efficient manner. However, these innovative strategy formulation approaches can prove extremely problematic, since many components of water systems possess extensive stochastic uncertainties. The prevalence of stochastic uncertainty renders most common decision-making approaches relatively unsuitable for practical WRM implementation.

Since water systems possess all of the characteristics connected with environmental planning, WRM has generally provided an ideal background for testing a wide range of decision support techniques used in environmental decision-making [3-5]. WRM decision-making frequently possesses incompatible and inconsistent design provisions that prove challenging to formulate into standard mathematical models [1-6]. This situation commonly occurs when final decisions must be constructed based not only upon clearly articulated specifications, but also upon environmental, political and socio-economic objectives.
that are either fundamentally subjective or not clearly articulated [7-10]. While “Optimal” results can be explicitly calculated for the mathematical formulations, whether these represent the truly best courses-of-action to pursue for the “real” problem remains somewhat tentative. Moreover, in public policy formulation, it may never be possible to explicitly convey many of the subjective considerations because there are numerous competing, adversarial stakeholder groups holding diametrically opposed perspectives. Therefore, many of the subjective aspects remain unknown, unquantified and unmodelled in the construction of any corresponding decision models. WRM policy determination can become even more complex when the various system components contain considerable stochastic uncertainties [10]. Consequently, WRM policy formulation can be considered an extremely complicated and challenging enterprise [10,11].

Within WRM decision-making, there are routinely many stakeholder groups holding incongruent viewpoints, essentially dictating that policy-makers need to construct decision frameworks that can somehow simultaneously reflect numerous irreconcilable viewpoints. In such situations, it is often more desirable to create a small number of dissimilar alternatives that provide dissimilar viewpoints for the particular problem [3,9]. These distinct solutions should be near-optimal when measured with respect to the quantified objective(s), but be maximally different from each other within the decision space. Several techniques collectively referred to as Modelling-to-Generate-Alternatives (MGA) have been created to address this multi-solution [6,7,9]. The fundamental impetus underlying MGA is to construct a set of alternatives that are “Good” with respect to the Stated objective(s), but fundamentally different from each other in the decision region. Decision-makers must then conduct a subsequent assessment of the alternatives to determine which Specific alternative(s) most closely achieve their specific goals. Therefore, MGA approaches are classified as decision support processes instead of solution creation methods as assumed in optimization.

The earliest MGA methods employed basic, incremental processes for creating their alternatives by iteratively re-running their procedures whenever a new solution needed to be constructed [6-10]. These iterative approaches replicated the seminal approach of Brill et al. [8] where, once an initial mathematical formulation had been optimized, all supplementary alternatives were produced one-at-a-time. These methods all employed \( n+1 \) iterations of their respective algorithms – initially to optimize the original problem, then to produce each of the \( n \) subsequent alternatives [9,11-18].

In this paper, it is demonstrated how a set of maximally different alternatives can be created by extrapolating several earlier MGA approaches into stochastic optimization [12-18]. The stochastic MGA process provides a method that can be deployed using any population-based solution algorithm. This process extends several earlier concurrent approaches [13-17] by permitting the simultaneous generation of \( n \) distinct alternatives in a single computational run. Consequently, to generate \( n \) maximally different alternatives, the algorithm runs exactly once irrespective of the value of \( n \) [19-23]. This data structure permits the solution generalization to all population-based solution algorithms. A multicriteria objective is employed that combines a novel data structure into the simultaneous solution approach to create an effective MGA approach. The use of this data structure permits the solution generalization to all population-based methods. Consequently, this stochastic, multicriteria MGA algorithmic approach proves to be extremely computationally efficient. The efficacy of this method for creating water resource alternatives is demonstrated by extending the MGA procedure to the “Real world” WRM optimization case study taken from [24,25].

Modelling to Generate Alternatives

Mathematical programming has focused almost exclusively on the construction of single optimal solutions to single-objective formulations or calculating sets of no inferior solutions for multi-objective models [2,5,7]. Although these techniques supply resolutions to the mathematical models, whether these outputs are truly best for the underlying “Real” problems is less obvious [1,2,6,7]. Within most “Real world” decision-making environments, there are countless system requirements and objectives that will never be explicitly apparent or included in the model formulation stage [1,5]. In addition, most subjective components unavoidably remain unquantified and unmodelled in the models constructed. This regularly occurs where final decisions are constructed based not only on modelled objectives, but also on more subjective socio-political-economic preferences and stakeholder goals [9]. Several incongruent modelling conditions are discussed in [6-8,10].

When unmodelled objectives and unquantified issues exist, non-traditional methods are needed to search the decision region for not only noninferior sets of solutions, but also alternatives that are plainly \textit{sub-optimal} for the modelled problem. Namely, any search for alternatives to problems known or suspected to contain unmodelled components must concentrate not only on a non-inferior set of solutions, but also necessarily on an explicit exploration of the problem’s inferior solution space.

To illustrate the consequences of unquantified objectives, suppose that the optimal solution for a maximization problem is \( X^* \) with objective value \( Z^* \) [26]. Assume an unmodelled second maximization objective \( Z_2 \) exists that epitomises some “Politically acceptable” factor. Suppose that a 2-objective noninferior solution, \( X^*_2 \), exists representing the best compromise solution if both objectives could have been considered concurrently. While \( X^*_2 \) would be the best solution to the real problem, in the mathematical model it would be considered inferior to \( X^* \) because \( Z_1^* \leq Z_1^* \).
Thus, when unquantified components can somehow be incorporated into the decision-making process, mathematically inferior solutions could prove optimal to the underlying “Real” problem. Therefore, when unmodelled issues and unquantified objectives might exist, mathematically “Different” solution approaches are needed to not only search the decision domain for noninferior solutions, but also to simultaneously explore the decision domain for inferior solutions. Population-based search algorithms permit coordinated explorations throughout a solution space and prove to be particularly adept solution procedures.

The purpose of MGA is to produce a practicable set of alternatives that are quantifiably good with respect to all modelled objectives, yet as distinct from each other as possible within the decision region. In accomplishing this task, the resultant set of alternatives supplies truly different perspectives that perform similarly with respect to the known modelled objective(s) yet very differently with respect to potentially unmodelled aspects. By generating such good-but-different alternatives, the decision-makers can then explore potentially desirable qualities within the solutions that might satisfy unmodelled objectives to varying degrees of stakeholder acceptability.

A formal mathematical representation is needed to characterize the MGA process [6,9]. Let the optimal solution to the original mathematical model be $X^*$ with corresponding objective value $Z^* = F(X^*)$. A subsequent difference model can then be solved that produces an alternative solution, $X$, which can be considered maximally different from $X^*$:

Maximize $\Delta(X, X^*) = \min_i |X_i - X^*_i|$ \hspace{1cm} (1)

Subject to: $X \in D$ \hspace{1cm} (2)

$|F(X) - Z^*| \leq T$ \hspace{1cm} (3)

where $\Delta$ denotes an appropriate difference function (shown in (1) as an absolute difference) and $T$ represents some targeted tolerance relative to the original optimal objective $Z^*$. $T$ is a user-specified limit that regulates what proportion of the inferior region will be explored for satisfactory alternatives. The difference concept can be extrapolated to a difference function between sets of alternatives by replacing $X^*$ in the objective of the maximal difference model and calculating the overall minimum absolute difference (or some other function) between the pairwise comparisons of corresponding variables in each pair of alternatives – subject to the requirement that each alternative is feasible and falls within the specified tolerance constraint.

The subsequent population-based MGA procedure constructs sets of near-optimal, maximally different alternatives, by selectively altering the value of $T$ and solving the resulting maximal difference problem by exploiting the population structure of the algorithm. Solution survival is contingent upon how well the solutions perform with respect to the problem’s originally modelled objective(s) and simultaneously by how far away they are from all of the other alternatives generated in the decision space.

**Simulation-Optimization for Stochastic Optimization**

Optimizing large problems is difficult when numerous stochastic uncertainties need to be combined directly into the solution procedures [26-29]. Simulation-Optimization (SO) is a widely categorized family of solution approaches that combines simulation with optimization for stochastic optimization [26]. In SO, simulation models in which the decision variables provide the settings under which simulation is performed replace all unknown objective functions, constraints, and parameters.

The fundamental phases of SO can be stated in the following manner [28,30]. Suppose an optimization problem contains $n$ decision variables, $X_i$, represented by $X = [X_1, X_2, \ldots , X_n]$. If the objective function is designated by $F$ and the feasible region is expressed as $D$, then the problem is to optimize $F(X)$ subject to $X \in D$. Under stochastic conditions, the objective and constraint values can be calculated via simulation. A comparison between any two solutions $X1$ and $X2$ involves the calculation of some statistic of $F$ modelled with $X1$ compared to the same statistic modelled with $X2$ [26,31]. These statistics are determined in a simulation model where each $X$ provides the decision variable settings. While simulation can be used to compare results, it cannot determine the optimal solutions. Hence, stochastic optimization cannot be accomplished autonomously through simulation alone.

As all system measures in SO are stochastic, each solution, $X$, needs to be estimated by simulation. Since simulation is computationally intensive, an optimization component is required to direct the search through the problem’s solution domain using as few simulation runs as possible [29,31]. Because most stochastic problems possess numerous potential solutions, in general, extensive solution searches need to be made through all portions of the feasible domain. Hence, a stochastic SO method contains two alternating computational phases [32]. In phase 1, an “Evolutionary” component directed by some optimization method is employed. In phase 2, the solution values from phase 1 are calculated via simulation. Because of the stochastic components, all performance measures are statistics determined by the responses generated in the simulation phase. Namely, solution quality is found by the value of the performance criterion, $F$, determined in the simulation phase. After simulating each candidate solution, the corresponding objective values are returned to the evolutionary phase in order to construct the ensuing candidate solutions. Thus, the evolutionary phase directs the search toward improved solutions in subsequent generations by ensuring that the solution search does not become
stuck at local optima. After constructing new candidate solutions in the evolutionary phase, the new solution set is sent back to the simulation phase for comparative evaluation. The two-phase search process terminates when an appropriately static system state has been attained (i.e. an optimal solution). The optimal solution is the single best solution found throughout the entire search process [32].

It has been demonstrated that SO can be used as a very computationally intensive, stochastic MGA technique [31,33]. However, because of the very long computational runs, several approaches to accelerate the search times and solution quality of SO have been examined subsequently [30]. Population-based algorithms prove beneficial in SO as the solution set contained within their populations allow for simultaneous searches through multiple sections of the feasible region. A primary characteristic of population-based algorithms is that better solutions in a current population possess a greater likelihood for survival and progression into the subsequent population. The next section provides a population-based MGA algorithm that incorporates stochastic uncertainty using SO to much more efficiently generate solutions of maximally different solution alternatives.

**Multicriteria Population-Based MGA Computational Algorithm**

In this section, a data structure is used to enable multicriteria MGA via any population-based algorithm [34-36]. Suppose that the aim is to produce $P$ different alternatives with each solution consisting of $n$ decision variables. The entire population contains $K$ solutions in total. Thus, each solution in the population consists of one entire set of $P$ maximally different alternatives. Let $Y_k$, $k = 1,\ldots, K$, represent the $k^{th}$ solution which consists of one complete set of $P$ different alternatives. Specifically, if $X_{kp}$ corresponds to the $p^{th}$ alternative, $p = 1,\ldots, P$, of solution $k$, $k = 1,\ldots, K$, then $Y_k$ can be written as

$$Y_k = [X_{k1}, X_{k2}, \ldots, X_{kP}].$$

If $X_{kp}$, $q = 1,\ldots, n$, is the $q^{th}$ variable in the $j^{th}$ alternative of solution $k$, then

$$X_{kj} = [X_{kj1}, X_{kj2}, \ldots, X_{kjP}].$$

Therefore, the entire population, $Y$, consisting of $K$ different sets of $P$ alternatives, can be written as the vector,

$$Y' = [Y_1, Y_2, \ldots, Y_K].$$

The following multicriteria population-based MGA algorithm produces a pre-determined number of near-optimal, but maximally different alternatives, by modifying the value of the bound $T$ in equation (3) of the maximal difference model and using a population-based method to solve the corresponding, maximal difference problem. Each solution, $Y_k$, $k = 1,\ldots, K$, in the population encompasses one entire set of $P$ different alternatives. By exploiting the co-evolutionary characteristics of the algorithm, the procedure evolves each solution, $Y_k$, toward sets of dissimilar local optima within the solution space. In the procedure, each solution alternative, $Y_k$, undergoes the search steps of the algorithm. Solution survival depends upon both upon how well the solutions perform with respect to the modelled objective(s) and by how far apart they are from every other alternative in the decision domain.

A simple course for generating alternatives incrementally solves the maximum difference model by iterating the value of the target $T$ when a new alternative needs to be formed and then re-solving the resulting model [34]. This straightforward MGA approach matches the seminal Hop, Skip, and Jump (HSJ) algorithm [7] where the alternatives are created one-at-a-time through the incremental adjustment of the target constraint. While this approach is straightforward, it entails a repetitive execution of the optimization algorithm [9,12,13]. To advance the HSJ process, Imanirad et al. [13-15] designed a concurrent MGA approach employing co-evolution. In the co-evolutionary technique, pre-specified stratified subpopulation ranges within an algorithm’s overall population are established that collectively evolve the search toward the specified number of maximally different alternatives. Each desired solution alternative is represented by each respective subpopulation and each subpopulation undergoes the common processing operations of the procedure. The survival of solutions in each subpopulation depends simultaneously upon how well the solutions perform with respect to the modelled objective(s) and by how far away they are from all of the other alternatives. Thus, the evolution of solutions in each subpopulation toward local optima is directly influenced by those solutions contained in all of the other subpopulations, which forces the concurrent co-evolution of each subpopulation towards good but maximally distant regions within the decision space according to the maximal difference model [9]. Co-evolution is also much more efficient than a sequential HSJ-style approach in that it exploits the inherent population-based searches to concurrently generate the entire set of maximally different solutions using only a single population [12,16].

While concurrent approaches can exploit population-based procedures, co-evolution only occurs in each stratified subpopulation. Consequently, the maximal differences between solutions in different subpopulations must depend on aggregated subpopulation measures. Conversely, in the subsequent algorithm, each solution in the population consists of an entire set of alternatives and maximal difference is calculated only for that specific solution (i.e. the particular set of alternatives contained within that
specific solution of the population). Hence, by the evolutionary characteristics of population-based search procedure, in the subsequent approach, the maximal difference is simultaneously calculated for the specific set of alternatives considered within each specific solution - and the need for concurrent subpopulation aggregation measures is circumvented.

Using the data structure terminology, the steps for the multicriteria population-based, stochastic MGA algorithm are as follows (see also: [18-23,34-38]). It should be readily apparent, however, that the stratification approach employed by this method can be easily modified to enable solution by any population-based algorithm [37].

Initialization Step. Solve the original optimization problem to find its optimal solution, \( X^* \). Based upon the objective value \( F(X^*) \), establish \( P \) target values. \( P \) represents the desired number of maximally different alternatives to be generated within prescribed target deviations from \( X^* \). Note: The value for \( P \) has to have been set a priori by the decision-maker.

Without loss of generality, it is possible to forego this step and to use the algorithm to find \( X^* \) as part of its solution processing in the subsequent steps. However, this can significantly increase the number of iterations of the computational procedure and the initial stages of the processing become devoted to finding \( X^* \) while the other elements of each population solution are retained as essentially “computational overhead”.

Step 1: Create an initial population of size \( K \) where each solution contains \( P \) equally-sized partitions. The partition size corresponds to the number of decision variables in the original optimization problem. \( X_{kp} \) represents the \( p^{th} \) alternative, \( p = 1, \ldots, P \), in solution \( Y_{kp}, \ k = 1, \ldots, K \).

Step 2: In each of the \( K \) solutions, evaluate each \( X_{kp}, \ p = 1, \ldots, P \), using the simulation module with respect to the modelled objective. Alternatives meeting their target constraint and all other problem constraints are designated as feasible, while all other alternatives are designated as infeasible. A solution can only be designated as feasible if all of the alternatives contained within it are feasible.

Step 3: Apply an appropriate elitism operator to each solution to rank order the best individuals in the population. The best solution is the feasible solution containing the most distant set of alternatives in the decision space (the distance measures are defined in Step 5).

Note: Because the best solution to date is always retained in the population throughout each iteration, at least one solution will always be feasible. A feasible solution for the first step can always consist of \( P \) repetitions of \( X^* \).

Step 4: Stop the algorithm if the termination criteria (such as maximum number of iterations or some measure of solution convergence) are met. Otherwise, proceed to Step 5.

Step 5: For each solution \( Y_{kp}, \ k = 1, \ldots, K \), calculate \( R \) Max-Min and/or Max-Sum distance measures, \( D_{k1}^{r}, r = 1, \ldots, R \), between all of the alternatives contained within the solution.

As an illustrative example for calculating the multicriteria distance measures, compute

\[
D_{k}^{1} = \Delta_{1}^{1} (X_{a}^{k}, X_{b}^{k}) = \text{Min}_{a \neq b} \left| X_{a}^{k} - X_{b}^{k} \right|, \quad a = 1, \ldots, P, \ b = 1, \ldots, P, \ q = 1, \ldots, n, \quad (7)
\]

\[
D_{k}^{2} = \Delta_{2}^{1} (X_{a}^{k}, X_{b}^{k}) = \sum_{p=1}^{P} \sum_{q=1}^{n} \left| X_{p}^{k} - X_{q}^{k} \right|, \quad (8)
\]

and

\[
D_{k}^{3} = \Delta_{3}^{1} (X_{a}^{k}, X_{b}^{k}) = \sum_{p=1}^{P} \sum_{q=1}^{n} \left( X_{p}^{k} - X_{q}^{k} \right)^{2}, \quad (9)
\]

\( D_{k}^{1} \) denotes the minimum absolute distance, \( D_{k}^{2} \) represents the overall absolute deviation, and \( D_{k}^{3} \) represents the overall quadratic deviation between all of the alternatives contained within solution \( k \).

Alternatively, the distance functions could be calculated by some other appropriately defined function.

Step 6: Let \( D_{k} = G(D_{k}^{1}, D_{k}^{2}, D_{k}^{3}, \ldots, D_{k}^{R}) \) represent the multicriteria objective for solution \( k \). Rank the solutions according to the distance measure \( D_{k} \) objective – appropriately adjusted to incorporate any constraint violation penalties for infeasible solutions. The goal of maximal difference is to force alternatives to be as far apart as possible in the decision space from the alternatives of each of the partitions within each solution. This step orders the specific solutions by those solutions which contain the set of alternatives which are most distant from each other.

Step 7: Apply applicable algorithmic “Change operations” to each solution within the population and return to Step 2.

Water Resources Management Case Study

Water resources problems often contain significant stochastic uncertainties. Under such circumstances, WRM decision-makers prefer to select from a set of contrasting, “near best” alternatives that differ from each other in terms of the system structures provided by their decision variables. The efficacy of the multicriteria population-based MGA procedure will be illustrated using a WRM case taken from [24,25]. While this section briefly summarizes the case, more explicit details, data, and descriptions can be found in [24,25,36,39-42].

Earlier studies [24,25] considered a WRM problem for allocating water in a dry season from an unregulated reservoir to three distinct user classes: (i) An industrial concern, (ii) an agricultural sector, and (iii) a municipality (see Figure 1). The industrial and agricultural segments were in a period of significant capital expansion and wanted to establish the water allocation
levels to which they were entitled. If insufficient water quantities were available, these sectors would be forced to curb their planned expansions. If adequate supplies of water could be delivered, this would contribute positive net benefits to the local economy. However, if sufficient water could not be guaranteed, net benefits provided by the users would be curtailed.

**Figure 1:** Schematic of Water-Allocation to Multiple Users.

In the region, the water demands have been increasing temporally due to economic expansion and population growth. Because of this, it is necessary to establish where the different water users stand by supplying sufficient information to make decisions regarding various investments and activity levels. For example, industries are not likely to develop water intensive projects if they know, a priori, that water consumption will need to be limited. Similarly, farmers aware that there is only a small chance of receiving sufficient water during a dry season are unlikely to undertake major investments in irrigation infrastructure. If needed water cannot be delivered due to insufficiency, the users will have to either curtail their development plans or obtain water from more expensive alternate sources. (Tables 1 and 2) show the related water resources and economic data [24,25]. For example, municipal residents may have to restrict the watering of lawns, industries may have to increase water recycling rates or reduce production levels, and farmers may not be able to irrigate as planned. These impacts will result in increased costs or decreased benefits in relation to the regional development. Thus, it is necessary that the available water be effectively allocated to minimize any associated penalties. The problem can be formulated as maximizing the expected net system benefits. Based upon local water management policies, a quantity of water can be pre-defined for each user. If this quantity is delivered, it will result in net benefits; however, if it cannot be delivered, the system will be subject to some form of penalties.

<table>
<thead>
<tr>
<th>Activity / User</th>
<th>Municipal ((i = 1))</th>
<th>Industrial ((i = 2))</th>
<th>Agricultural ((i = 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum allowable allocation (W_{i_{\text{max}}}^{\pm})</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

**Table 1** Allowable Water Allocations (m$^3$ Millions) and Related Economic Data ($ Millions)

<table>
<thead>
<tr>
<th>Activity / Flow level</th>
<th>Low ((j = 1))</th>
<th>Medium ((j = 2))</th>
<th>High ((j = 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seasonal flow rate (q_j^{\pm})</td>
<td>[3.2, 4.8]</td>
<td>[7, 13]</td>
<td>[14, 20]</td>
</tr>
<tr>
<td>Probability (p_j)</td>
<td>0.2</td>
<td>0.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table 2** Streamflow Distribution (m$^3$ Millions) and the Associated Probabilities.

Consequently, under the conditions of uncertainty, the primary decisions are: (i) how to efficiently apportion water to each of the three sectors to maximize the anticipated net benefits; and, (ii) how to establish the water policies (in terms of allowable amounts) with the least risk of system disruption. Within these decisions is a determination of which one of the many pathways that the water would need to flow through to reach the users. In addition, it is possible to subdivide the various water streams, with each substream sent to a different user. Since cost differences from operating the facilities at different capacity levels produce economies of scale, decisions have to be made to determine how much water should be sent along each flow pathway to each user type. Thus, any individual policy entails a combination of many decisions regarding which facilities were allocated water and the quantities of water that would be directed to each user type. These decisions are further complicated by prevailing system uncertainties vis-à-vis the seasonal water flows together with their corresponding likelihoods.
The WRM authority is responsible for allocating water to each of the municipality, the industrial concerns, and the agricultural sector. As the quantity of stream flows from the reservoir are uncertain, the problem is formulated as a stochastic programming problem. This stochastic programming model can account for the uncertainties in water availability. However, uncertainties may also exist in other parameters such as benefits, costs and water-allocation targets. In the formulation, penalties are imposed when policies that have been expressed as targets are violated. Also, within the model, any uncertain parameter \( A \) is represented by \( A^\pm \) and its corresponding values are generated via probability distributions. To reflect all of these uncertainties, the following stochastic programming model was constructed by [25]:

\[
\text{Max } f^+ = \sum_{i} B_i^+ W_i^+ - \sum_{i} \sum_{j} p_j S_j^i \\
\text{Subject to:} \\
\sum_{i} (|W_i^+ - W^\text{max}_i| - S_j^i) \leq q_j^+ \quad \forall j \\
S_j^i \leq W_i^+ \leq W^\text{max}_i \quad \forall i \\
S_j^i \geq 0 \quad \forall i, j
\]  

In this formulation \( f^+ \) represents the net system benefit ($/m^3$) and \( B_i^+ \) represents the net benefit to user \( i \) per m$^3$ of water allocated ($$). \( W_i^+ \) is the fixed allocation amount (m$^3$) for water that is promised to user \( i \), while \( W^\text{max}_i \) is the maximum allowable amount (m$^3$) that can be allocated to user \( i \). The loss to user \( i \) per m$^3$ of water not delivered is given by \( C_i^+ \), where \( C_i > B_i ($) \). \( S_j^i \) corresponds to the shortage of water, which is the amount (m$^3$) by which \( W_j \) is not met when the seasonal flow is \( q_j \). \( q_j^+ \) is the amount (m$^3$) of seasonal flow with \( p_j \) probability of occurrence under flow level, where \( p_j \) provides the probability (%) of occurrence of flow level \( j \). The variable \( i = 1, 2, 3 \), designates the water user, where \( i = 1 \) for municipal, 2 for industrial, and 3 for agricultural. The value of \( j = 1, 2, 3 \), is used to delineate the flow level, where \( j = 1 \) represents low flows, 2 represents medium flows, and 3 represents high flows. Finally, \( m \) is the total number of water users and \( n \) is the total number of flow levels.

Using this stochastic formulation, the WRM problem is to effectively assign the water to the three user groups to maximize the overall net benefits and to incorporate water policies in terms of determining permissible amounts with the least risk for causing system disruption. By representing the stochastic uncertainties as probability distributions, interval estimates, and uncertainty membership functions, Maqsood et al. [25] constructed a solution to the WRM problem with an overall net benefit of $2.02 million.

While knowing the optimal solution to the mathematical model establishes an important quantitative benchmark, WRM planners faced with difficult and potentially controversial choices generally tend to prefer to choose from a set of near-optimal alternatives that differ significantly from each other in terms of their system structures. To create such planning options for the WRM system, one approach would be to place extra target constraints into the mathematical model to force solutions that were different from the one initially determined by optimization. For example, suppose that five additional planning alternative options were created through the inclusion of a technical constraint on the objective function that decreased the total system benefits of the original model from 2% up to 10% in increments of 2%. By adding these incremental target constraints to the original SO model and sequentially re-solving the problem an additional 5 times, it would be possible to construct the specific number of alternative policies for WRM planning.

Rather than running five additional, separate instances of the computationally intensive SO procedure to create these solutions, the multicriteria population-based MGA algorithm from the previous section was run a single time to produce the requisite additional alternatives. (Figure 2) shows a schematic layout showing how the MGA approach is applied to the WRM case and (Table 3) provides the overall system benefits for the 5 maximally different options generated.

![Figure 2: Schematic of the Multicriteria MGA Methodology Applied to WRM Case.](image-url)
Table 3: System Benefits ($ Millions) for 6 Maximally Different Alternatives.

<table>
<thead>
<tr>
<th>Maximally Different Solutions</th>
<th>WRM System Benefits ($ Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Solution Overall</td>
<td>2.02</td>
</tr>
<tr>
<td>Best Solution Within 2%</td>
<td>1.99</td>
</tr>
<tr>
<td>Best Solution Within 4%</td>
<td>1.96</td>
</tr>
<tr>
<td>Best Solution Within 6%</td>
<td>1.91</td>
</tr>
<tr>
<td>Best Solution Within 8%</td>
<td>1.86</td>
</tr>
<tr>
<td>Best Solution Within 10%</td>
<td>1.79</td>
</tr>
</tbody>
</table>

(Table 4) shows the actual water allocation values obtained for the overall best solution determined under optimized water-allocation targets. It can be observed that solutions for the objective function value and most of the non-zero decision variables related to the agricultural and industrial water can occur over an interval range, while those related to municipal water use are explicit deterministic values. In case of insufficient water supply, allocation should firstly be guaranteed to the municipality, secondly to the industry, and lastly to the agriculture. This is because municipal use provides the highest benefit when water demand is satisfied and is subject to the highest penalty if the promised water is not delivered; whereas, the industrial and agricultural uses correspond to lower benefits and penalties.

Table 4: Best Water Allocations Under Optimized Water-Allocation Targets (m$^3$ Millions).

<table>
<thead>
<tr>
<th>Activity/ User</th>
<th>Municipal $(i = 1)$</th>
<th>Industrial $(i = 2)$</th>
<th>Agricultural $(i = 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target $(W^\pm_j)$</td>
<td>2.5</td>
<td>4.0</td>
<td>4.6</td>
</tr>
<tr>
<td>Shortage $(S^\pm_j)$ under a flow level of:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low $(j = 1)$</td>
<td>0</td>
<td>[2.1, 2.7]</td>
<td>4.6</td>
</tr>
<tr>
<td>Medium $(j = 2)$</td>
<td>0</td>
<td>0</td>
<td>[0, 1.7]</td>
</tr>
<tr>
<td>High $(j = 3)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Allocation (under a flow level of:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low $(j = 1)$</td>
<td>2.5</td>
<td>[1.3, 1.9]</td>
<td>0</td>
</tr>
<tr>
<td>Medium $(j = 2)$</td>
<td>2.5</td>
<td>4.0</td>
<td>[2.9, 4.6]</td>
</tr>
<tr>
<td>High $(j = 3)$</td>
<td>2.5</td>
<td>4.0</td>
<td>4.6</td>
</tr>
<tr>
<td>Net benefit ($ Millions)</td>
<td>$f^\pm = 2.02$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Table 5) contrasts alternative options under different water-allocation schemes. The best solution within 2% alternative would correspond to the situation in which the WRM manager is essentially optimistic regarding water availability. Thus, a plan with both higher shortage and higher allocation is generated, where risks of water insufficiency may exist but the net benefit is quite considerable. This leads to a plan with both lower shortage and lower allocation, but a higher risk of wasting available water. For the best solution within 6%, the resulting plan will be effective in high-runoff years when all targeted water demands can be delivered, but becomes riskier under low-runoff conditions due to the deficit of water availability and the relevant penalties. The solution corresponds to a situation in which water availability falls somewhere in-between conservative and optimistic conditions. Conversely, the best solution within 10% represents a situation in which the manager is pressured to be conservative regarding the
water availability and, therefore, must allocate essentially lower bound target values to all users.

<table>
<thead>
<tr>
<th></th>
<th>Best Solution Within 2%</th>
<th>Best Solution Within 6%</th>
<th>Best Solution Within 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Target</strong> ( W_i^\pm )</td>
<td>Municipal</td>
<td>Industrial</td>
<td>Agricultural</td>
</tr>
<tr>
<td>1.5</td>
<td>2.0</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td><strong>Shortage</strong> ( S_f^\pm ):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( j =1 )</td>
<td>0</td>
<td>0</td>
<td>[2.6, 2.94]</td>
</tr>
<tr>
<td>( j =2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( j =3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Allocation</strong> ( A_f^\pm ):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( j =1 )</td>
<td>1.5</td>
<td>2.0</td>
<td>[0.56, 0.9]</td>
</tr>
<tr>
<td>( j =2 )</td>
<td>1.5</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>( j =3 )</td>
<td>1.5</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td><strong>Net Benefit</strong> ( $ \text{ Millions} )</td>
<td>$1.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Alternative Solutions Under Different Water-Allocations \((\text{m}^3 \text{ Millions})\).

Irrespective of the final decision, given the performance bounds established for each problem alternative, the WRM policymakers can be confident in the stated performance objective of each option while simultaneously recognizing that they are as structurally different from each other as feasibly possible. Therefore, if there might be stakeholders holding diametrically opposing standpoints, the decision-makers can contrast the options without being myopically constrained by any dominant perspective based exclusively on objective value.

The WRM case illustrates several important characteristics with respect to the multicriteria MGA algorithm: (i) Population-based procedures can be effectively used as the fundamental optimization search method in SO routines; (ii) Population-based MGA solution searches can concurrently create more good alternatives than planners could construct using other approaches; (iii) Because of the structure of the MGA algorithm, the alternatives generated are good for planning purposes since all of their solutions are guaranteed to be as maximally different from each other as possible; (iv) Because the procedure needs to run only once to generate its entire set of multiple, good solution alternatives, it is very computationally efficient (i.e. the algorithm runs exactly once to generate \( n \) maximally different solution alternatives, irrespective of the value of \( n \)); and, (v) The best overall solution produced by the multicriteria MGA procedure will be identical to the best solution produced by function optimization alone.

Conclusions

Water resource management problems contain multidimensional performance specifications which inevitably include incongruent performance components and unquantifiable modelling features. These problems often possess incompatible design specifications which are impossible to completely incorporate into the supporting decision models. Consequently, there are unmodelled problem components, generally not apparent during model construction, that can significantly influence the acceptability of any model’s solutions. These competing and ambiguous components force WRM decision-makers to incorporate many conflicting requirements into their decision process prior to the final solution determination. Consequently, water management decision-makers generally prefer to select from a set of distinct planning perspectives.

This paper has applied a stochastic, multicriteria population-based MGA procedure to WRM planning. This computationally efficient MGA approach establishes how population-based
algorithms can simultaneously construct entire sets of near-optimal, maximally different alternatives by exploiting the evolutionary characteristics of population-based solution algorithms. In this MGA role, the multicriteria objective can efficiently generate the requisite set of dissimilar alternatives, with each generated solution providing an entirely different perspective to the problem. Since population-based procedures can be applied to a wide range of problem types, the practicality of this multicriteria MGA approach can be extended to wide array of “real world” environmental applications beyond WRM settings. These extensions will be considered in future studies.

**Acknowledgement**

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**References**


